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NUMERICAL ANALYSIS OF FRAMES WITH CURVED GIRDERS

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ENGINEERING MECHANICS DIVISION

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NUMERICAL ANALYSIS OF FRAMES WITH CURVED GIRDERS

James Michalos*, Member ASCE

March, 1953

SYNOPSIS

An arithmetical procedure is presented for determining moments in continuous frames having curved girders. Values of the required properties for application of the procedure in the usual case of girders of circular arc plan are included.

INTRODUCTION

In keeping with modern trends in architecture, and with clearance and space requirements, it seems likely we shall see an increasing use of frames with curved girders.

Girders of circular arc plan, such as that illustrated in Fig. 1(a), have been analyzed[1, 2] by distributing moments about normal and tangential axes at the supports. In (b) and (c) of Fig. 1 are shown plan views of frameworks in which the normal and tangential axes of the several members framing into a joint are not similar. Methods used for analyzing the structure in Fig. 1 (a) are not applicable to structures such as those in (b) and (c).

In a previous paper[3] the author presented a numerical procedure for determining moments in continuous frames whose members are straight but join at various angles in space. That procedure involves successive corrections to moments about arbitrarily chosen rectangular axes.

In this paper the same general idea is used, with all moments and properties of the members referenced to a pair of orthogonal axes in the plane of the frame. The members can have any curvature, including segmental, in plan, but of particular concern in this paper is the common, circular arc shape. Tables of the required properties of such shapes are included, as are numerical examples illustrating the procedure.

For open sections, such as I-beams and channels, it would be important to consider the effect of warping restraint when determining the torsion factor, J. However, such open sections would hardly be used for curved girders subjected to appreciable torques. For closed sections, as contemplated here-

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1. "Moment Distribution Applied to Flexure and Torsion", by Mervin B. Hogan, Bulletin No. 26, Eng. Experiment Station, Univ. of Utah, Salt Lake City, Utah, 1945.
2. "Analysis of Continuous Circular Curved Beams", by Becla Velutini, Journal, ACI, Nov. 1950.
3. "Numerical Analysis of Continuous Frames in Space", by James Michalos, presented at the ASCE Convention, Denver, June 19, 1952, and scheduled for publication in Proceedings, ASCE.

in, the warping effects are considered secondary and are not included.

STIFFNESSES AND CARRY-OVER FACTORS

In Fig. 2(a) a moment, M_{zB}, is applied about the z axis at end B of a curved member lying in the xz-plane. If no rotation is permitted about the x axis at the same end, and likewise no rotation is permitted about either the x or z axes at end A, the applied moment, M_{zB}, will induce moments about the x axis at B and about the x axis and z axis at A. These moments are labeled $M_{zB}\,(C_{xz})_{B\text{-}B}$, $M_{zB}(C_{xz})_{B\text{-}A}$, and $M_{zB}\,(C_{zz})_{B\text{-}A}$. The terms $(C_{xz})_{B\text{-}B}$, $(C_{xz})_{B\text{-}A}$, and $(C_{zz})_{B\text{-}A}$ are called carry-over factors. The first subscript within a parenthesis designates the axis to which the carry-over is made, and the second subscript designates the axis about which a moment was applied. The subscripts outside a parenthesis indicate the direction of carry-over.

In (b), (c), and (d) of Fig. 2 are shown the carry-over moments resulting from the application of moments M_{zA}, M_{xB}, and M_{xA}, respectively. All moments in Fig. 2 are positive, and are shown as vectors. Use the right-hand screw rule for sense of these positive moments.

Suppose that M_{zB} is of such a magnitude that the end B is rotated through a unit angle about the z axis while no rotation is allowed to take place about the x axis at B and about the x and z axes at A. Such a moment is defined as the stiffness about the z axis at end B and is labeled K_{zB}. Similarly, there are stiffnesses K_{zA}, K_{xB}, and K_{xA}.

Stiffnesses for various lengths and orientations of circular segments are given in Table 1. Values are tabulated for three ratios of bending to torsional rigidity (EI/GJ) along the segment. All values of stiffness given in the table are in terms of r/EI, where r is the radius of curvature. The method by which these stiffness values were determined is discussed at the end of this paper.

Curves can be drawn from the values given in Table 1, and one set of curves is shown in Fig. 3. Such curves enable one to interpolate between tabulated values and also between the three values of EI/GJ considered in making the tabulation. A study of Table 1 will show that the values of all four stiffnesses are related, and consequently all four stiffnesses can be read from one chart such as Fig. 3.

Carry-over factors are given in Table 2. By tabulating carry-over factors in terms of the stiffnesses, only six sets of values are required, for each ratio of EI to GJ, instead of twelve. This is because of the equalities resulting from the existing reciprocal relationships. All values in Table 2 are also in terms of r/EI. Therefore a carry-over factor can be obtained directly by dividing a value in Table 2 by the appropriate value from Table 1. The determination of the values in Table 2 is discussed at the end of the paper.

For general use, curves should be drawn by means of the values in Table 1. Actually, only three sets of curves need be constructed for each value of the ratio EI to GJ. For example, if a set of curves is drawn for determining $(C_{xz})_{A\text{-}B}$, values of $(C_{xz})_{B\text{-}A}$ can be obtained from the same curves by changing the sign of the angle η.

Curves for stiffnesses and for carry-over factors (in terms of stiffnesses) have the shape of sine, or cosine, curves. As a result, only a few points are necessary to adequately define any curve. In Tables 1 and 2, values are limited to an angle η equal to plus or minus 90 degrees, but since the shape of the curves is known, they can be extended to cover any value of η.

FIXED-END MOMENTS

In Fig. 4(a) positive bending and twisting moments are shown at the ends of a member AB. Positive moments about the x and z axes at the ends of the same member are shown in Fig. 4(b). These moments are related to the former in the following way:

$$\begin{aligned} m_{xA} &= m_t \sin\beta - m_b \cos\beta \\ m_{zA} &= m_t \cos\beta + m_b \sin\beta \\ m_{xB} &= m_t \sin\gamma + m_b \cos\gamma \\ m_{zB} &= m_t \cos\gamma - m_b \sin\gamma \end{aligned} \qquad \text{- - - - - - - - - - - - - - - (1)}$$

Table 3 contains values of fixed-end moments, resulting from a concentrated load W, for members of circular arc plan. Because of the more economical presentation which results, the fixed-end moments tabulated are those about the bending and twisting axes. For use in the numerical procedure of this paper, these bending and twisting moments are resolved into fixed-end moments about the x and z axes as indicated by Eqs. 1. The method by which the tabulated fixed-end bending and twisting moments were obtained is outlined at the end of this paper.

All values in Table 3 are in terms of the concentrated load, W, and the radius of curvature, r. If more than one concentrated load is on the member, add the fixed-end moments due to each one. For general use, curves for m_b and m_t should be drawn by means of the tabulated values. Such curves for m_b, with a ratio of EI to GJ equal to 2, are shown in Fig. 5. These curves have the typical shape of influence lines for fixed-end moment.

Values for fixed-end bending and twisting moments resulting from a uniformly distributed load are given in Table 4. Curves can be plotted for general use.

NUMERICAL EXAMPLES

A plan view of a frame containing curved members is shown in Fig. 6. The curved girders and straight beams are supported by columns at their intersections. At their opposite ends the straight beams are considered fixed. They could instead be framed into other members without affecting the numerical procedure except that it would be necessary to distribute carry-over moments at these ends.

From the dimensions and properties given in Fig. 6, the stiffnesses can be obtained from Table 1 for the curved members, and from 4EI/L and GJ/L for the straight members. They are as follows:

For members 1 - 2 and 3 - 4:-

$$K_{z1} = K_{z2} = 2.317\ EI/r = 2.317\ (4E/14.14) = 0.657\ E$$

$$K_{x1} = K_{x2} = 0.761\ EI/r = 0.761\ (4E/14.14) = 0.215\ E$$

For member 2 - 3:-

$$K_{z2} = K_{z3} = 2.317\ EI/r = 2.317\ (20E/28.28) = 1.642\ E$$

$$K_{x2} = K_{x3} = 0.761\ EI/r = 0.761\ (20E/28.28) = 0.539\ E$$

For the straight beams:-

$$K_x = 4EI/L = 4E\ (3/20) = 0.600\ E$$

$$K_z = GJ/L = EI/L = 3E/20 = 0.150\ E$$

For columns 1 and 4:-

$$K_x = 4EI_x/L = 4E\ (1/14) = 0.285\ E$$

$$K_z = 4EI_z/L = 4E\ (3/14) = 0.555\ E$$

For columns 2 and 3:-

$$K_x = 4EI_x/L = 4E\ (2/14) = 0.570\ E$$

$$K_z = 4EI_z/L = 4E\ (6/14) = 1.710\ E$$

For convenience in distributing unbalanced moments at joints 1, 2, 3 and 4, distribution factors equal to $K_x/\Sigma K_x$ and $K_z/\Sigma K_z$ were determined for each member at each joint. For example, at joints 1 and 4:

Member	K_x	K_z	Distribution Factors x - axis	Distribution Factors z - axis
1-2 (or 4-3)	0.215	0.657	0.195	0.395
1-5 (or 4-8)	0.600	0.150	0.546	0.090
Column	0.285	0.855	0.259	0.515
	1.100	1.662	1.000	1.000

In a similar manner, distribution factors were obtained for joints 2 and 3. All distribution factors (Dist. Factors) are listed near the top of Table 5.

Carry-over factors are obtained by means of the values listed in Table 2. For example, for member 1-2, with ϕ equal 90° and η equal to zero,

$$K_{z1}\ (C_{zz})_{1-2} = K_{z2}\ (C_{zz})_{2-1} = 1.044\ EI/r$$

Then, dividing by K_{z1} and K_{z2}, respectively, we obtain

$$(C_{zz})_{1-2} = \frac{1.044\ EI/r}{2.317\ EI/r} = 0.452$$

$$\text{and}\quad (C_{zz})_{2-1} = \frac{1.044\ EI/r}{2.317\ EI/r} = 0.452$$

Because the angles ϕ and η are the same for all three of the curved girders, the carry-over factors for all three are identical. The carry-over factors (C. O. Factors) are listed near the top of Table 5.

The fixed-end moments (F.E.M.) listed in Table 5 were obtained with the aid of Table 3. For all girders the ratio of ϕ_a to ϕ is equal to 0.5, and therefore the fixed-end bending moment and the fixed-end twisting moment at each end are, from Table 3,

$$m_b = 0.22171\ Wr$$

$$m_t = 0.01460\ Wr$$

Substituting the appropriate values of W and r in the above expressions, the moments m_b and m_t were found to be equal to 37.7 ft. kips and 2.5 ft. kips, respectively, for girders 1-2 and 3-4. For girder 2-3 these values were found to be 176.0 ft. kips and 11.6 ft. kips. Then placing these values in Eqs. 1, the fixed-end moments shown in Table 5 were obtained.

The columns and straight beams are not listed in Table 5 since it is not necessary to make intermediate distributions. Moments in these members are determined after the moments at the ends of the curved girders have been obtained.

All moments about x-axes are listed in the left half of Table 5, and all moments about z-axes in the right half. The unbalanced moments at each joint are distributed, by multiplying with the appropriate distribution factors, and listed opposite "Dist." For example, the original unbalance about the x axis at joint 2 is equal to the algebraic sum of -24.9 and -116.1, or -141.0. To balance this moment the distributions must be positive, and are

$$0.112\ (141.0) = +15.8 \text{ to member 2-1}$$
$$\text{and}\quad 0.280\ (141.0) = +39.5 \text{ to member 2-3}$$

The remainder of the unbalanced moment is taken by the column and member 2-6, and will be accounted for later.

The distributed moments are then multiplied by the appropriate carry-over factors and entered as carry-over moments. Carry-overs due to moments about x axes are listed opposite the designation C. O. M. (x), and carry-overs due to moments about z axes are listed opposite C. O. M. (z). To illustrate, consider the carry-over moments resulting from the initial distribution to the x axes. These moments are first multiplied by the upper line of carry-over factors — $(C_{xx})_{1-2}$ to $(C_{xx})_{4-3}$ — and the results are listed in the left-half opposite C. O. M. (x) as -15.8,-----, -15.8. They are listed in the left-half portion of the table because they are carry-over moments to the x axis, and they are placed opposite C. O. M. (x) because they are due to moments about the x axis. The moments distributed to the x axes are next multiplied by the carry-over factors in the middle line, $(C_{zx})_{1-2}$ to $(C_{zx})_{4-3}$.

The resulting products, -9.5,-----, +9.5, represent carry-over moments to z axes due to moments about x axes, and consequently are placed in the right-hand portion of the table, opposite the designation C. O. M. (x). Similarly, the use of the carry-over factors in the lowest line results in the values +2.9,-----, -2.9, placed in the right-hand portion of the table, and also placed opposite C. O. M. (x).

The carry-overs due to moments about z axes are then obtained as products of the carry-over factors and distributed moments in the right-hand section of the table. These carry-over moments are all entered opposite C. O. M. (z).

The next step is to determine the new unbalanced moment at each joint, about each axis, and to distribute it. For example, at joint 2 the unbalanced moment about the z axis is

$$+1.2 -3.5 -0.1 -8.3 +8.3 -3.5 = -5.9$$

This unbalance is distributed

$$0.158\ (5.9) = +0.9 \text{ to member 2-1}$$
$$\text{and } 0.394\ (5.9) = +2.3 \text{ to member 2-3}$$

Four cycles of carry-over and distribution were considered adequate in Table 5. Because of the symmetrical case considered, the final end moments about the x axis for member 2-3 are approaching the fixed-end moments, -116.1, and are listed as such. Also, the final end moments for member 1-2 are converging to a sum equal to the sum of the fixed-end moments, or 49.8.

The moments about x or z axes anywhere in the curved girders can now be found, by computations of statics, from the final end moments. Then moments about x and z axes can be resolved into bending and twisting moments.

The final step is to determine the moments in the columns and connecting straight beams. The final end moments obtained in Table 5 are shown in Fig. 7. The moments in the individual columns and straight beams can now be obtained by distribution of the moments acting on column and beam together. For example, the total moment to be distributed about the x axis to column and beam at joints 2 and 3 is 15.3 plus 116.1, or 131.4 ft. kips. This is distributed in proportion to their relative stiffnesses, 0.570 and 0.600, as follows:

$$\frac{0.570}{0.570 + 0.600}\ (131.4) = 64.0 \text{ to top of column}$$

$$\text{and } \frac{0.600}{0.570 + 0.600}\ (131.4) = 67.4 \text{ to the beam}$$

One-half of 64.0 and 67.4 is then carried over, respectively, to the far end of the column and the beam.

The total moment to be distributed about the z axis to column and beam at joint 2 is 59.0. The distribution is made

$$\frac{1.710}{1.710 + 0.150}\ (59.0) = 54.2 \text{ to top of column}$$

$$\text{and } \frac{0.150}{1.710 + 0.150}\ (59.0) = 4.8 \text{ to the beam}$$

where 1.710 and 0.150 represent the relative stiffnesses, about the z axis, of column and beam, respectively. One-half of 54.2 is carried-over to the

bottom of the column. However, since the moment 4.8 ft. kips is a torsional moment, all of it is carried over to the far end of the beam.

A plan view of a curved girder, fixed at its ends and with two intermediate, simple supports, is shown in Fig. 8. If columns and straight beams were framed into the supports, the numerical procedure would be no different except that distribution would be made to more than 2 members at a joint.

Stiffnesses and carry-over factors were obtained from Tables 1 and 2. Fixed-end bending moments and twisting moments due to uniform load were obtained by means of Table 4, and they were resolved into fixed-end moments about x and z axes.

Moments are distributed and carried-over in Table 6, in the same manner as before, but there is no distribution to, and no carry-over from, the fixed ends. Because of symmetry about the z axis in span 2-3, the final end moments about the x axis at joints 2 and 3 will converge to the fixed-end moments of span 2-3, and are listed as such.

DETERMINATION OF STIFFNESSES, CARRY-OVER FACTORS, AND FIXED-END MOMENTS

The stiffnesses, carry-over factors, and fixed-end moments listed in Tables 1, 2, 3, and 4 were obtained by means of the shear and torsion analogy[4]. Symbollic expressions were written for each of the terms in the three equations of the analogy. These were integrated to obtain general expressions for stiffnesses, carry-overs, and fixed-end moments which were then evaluated for the cases listed in Tables 1 to 4. This work was done in a thesis[5] prepared under the writer's direction. From the expressions given therein, values of stiffness, carry-over factors, and fixed-end moments can be obtained for any ratio of EI to GJ.

For members of other than circular plan and constant section, values of stiffnesses, carry-over factors, and fixed-end moments can be determined by numerical application of the shear and torsion analogy[4] in a manner similar to that used in obtaining, by means of the column analogy, such values for moment distribution in frames with haunched members.

CONCLUDING REMARKS

A numerical procedure has been presented for the structural analysis of continuous frames containing curved girders as well as straight members. The procedure is applicable whether the straight beams are parallel to one of the adopted axes, as in Fig. 6, or not. In the latter case, stiffnesses and carry-over factors for the straight beams can be obtained from a previous paper[3] by the writer. Moments are then distributed and carried over with respect to the adopted system of axes, as before.

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4. "Laterally Loaded Plane Structures and Structures Curved in Space", by Frank Baron and James Michalos, Transactions, ASCE, Vol. 117, 1952, p. 279.
5. "Properties for Numerical Analysis of Frames with Curved Girders", by Timothy Chin, a thesis submitted to the Graduate Faculty, Iowa State College, Ames, Iowa, in 1952, in partial fulfillment of the requirements for the degree Master of Science.

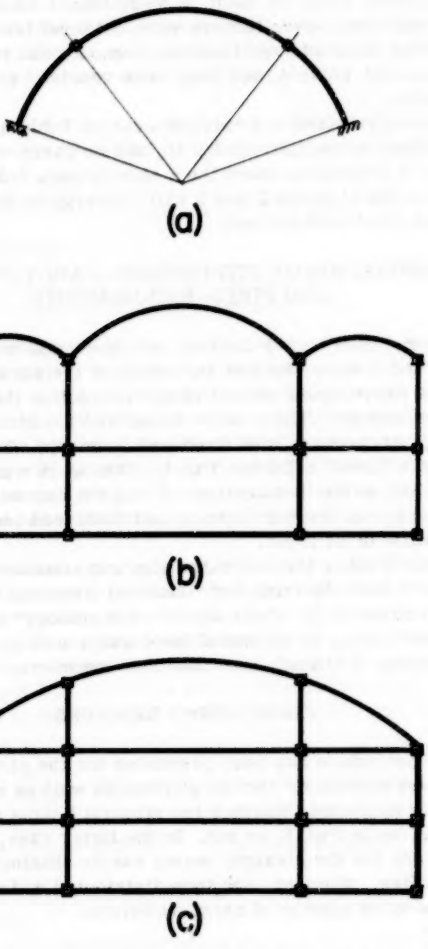


Fig. 1. Plan Views of Frames with Curved Members

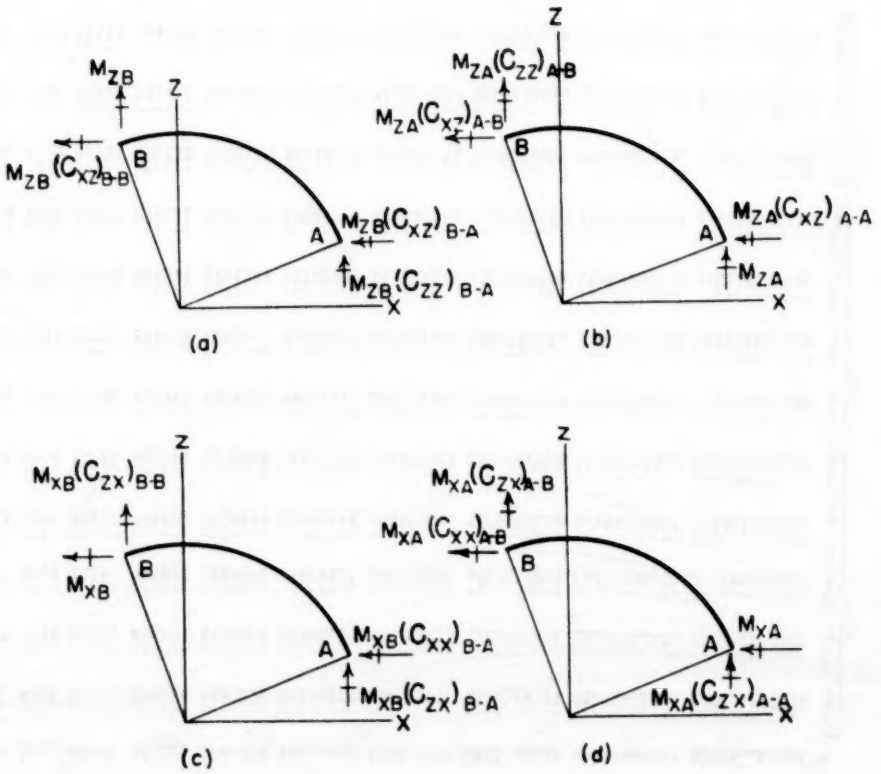
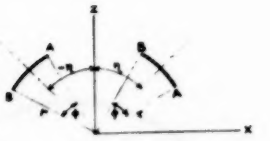


Fig. 2. Applied Moments and Resulting Carry-over Moments



		R1/R2 = 1				R1/R2 = 2				R1/R2 = 6			
ϕ	η	$K_{BB}(r/R1)$	$K_{BA}(r/R1)$	$K_{AB}r/R1)$	$K_{AA}(r/R1)$	$K_{BB}(r/R1)$	$K_{BA}(r/R1)$	$K_{AB}(r/R1)$	$K_{AA}(r/R1)$	$K_{BB}(r/R1)$	$K_{BA}(r/R1)$	$K_{AB}(r/R1)$	$K_{AA}(r/R1)$
	60	4.165	3.807	14.916	15.174	2.359	1.996	14.756	15.141	1.128	0.729	14.536	15.003
	45	2.863	3.158	13.318	13.803	4.425	3.372	12.711	13.766	3.446	2.167	12.278	13.558
	30	8.947	7.584	10.534	11.497	7.579	6.140	9.558	10.997	6.886	5.137	8.839	10.587
	15	11.497	10.534	7.584	8.947	10.997	9.558	6.140	7.579	10.587	8.839	5.137	6.886
	0	13.803	13.318	5.158	5.863	13.766	12.711	3.372	4.425	13.558	12.278	2.167	3.446
	60	13.803	13.318	5.158	5.863	13.766	12.711	3.372	4.425	13.558	12.278	2.167	3.446
15	45	15.174	14.916	3.807	4.165	15.141	14.756	1.996	2.359	15.003	14.536	1.129	0.729
	30	14.916	15.174	4.165	3.807	15.141	15.141	2.359	1.996	15.003	15.003	1.129	0.729
	15	13.218	13.803	5.863	5.158	12.711	13.766	4.425	3.372	12.278	13.558	3.446	2.167
	0	10.534	11.497	8.947	7.584	9.558	10.997	7.579	6.140	8.839	10.587	6.886	5.137
	-15	7.584	8.947	11.497	10.534	6.140	7.579	10.997	9.558	5.137	6.886	10.587	8.839
	-30	5.158	5.863	13.803	13.318	3.372	4.425	13.766	12.711	2.167	3.446	13.558	12.278
	-45	3.807	4.165	15.174	14.916	1.996	2.359	15.141	14.756	0.729	1.129	15.003	14.536
	75	2.577	2.081	6.938	7.411	1.853	1.123	6.638	7.368	1.367	0.476	6.255	7.126
	60	3.784	2.906	5.730	6.588	3.285	2.022	5.206	6.470	2.587	1.378	4.724	6.204
	45	5.253	4.260	4.260	5.253	4.975	3.516	3.516	4.975	4.672	2.929	4.672	4.672
	30	6.588	5.730	2.906	3.784	6.470	5.206	2.022	3.285	6.204	4.724	2.378	2.587
	15	7.411	6.938	2.081	2.577	7.368	6.638	1.123	1.853	7.126	6.255	0.476	1.367
30	0	7.541	7.541	1.953	1.953	7.411	7.411	1.068	1.068	7.138	7.138	0.465	0.465
	-15	6.938	7.411	2.577	2.081	6.638	7.368	1.853	1.123	6.255	7.126	1.367	0.476
	-30	5.730	6.588	3.784	2.906	5.206	6.470	3.285	2.022	4.724	6.204	2.587	1.378
	-45	4.260	5.253	5.253	4.260	3.516	4.975	4.975	3.516	2.929	4.672	4.672	2.929
	-60	2.906	3.784	6.588	5.730	2.022	3.285	6.470	5.206	1.378	2.587	6.204	4.724
	-75	2.081	2.577	7.411	6.938	1.123	1.853	7.368	6.638	0.476	1.367	7.126	6.255
	60	2.207	1.526	4.097	4.789	1.876	0.878	3.703	4.700	1.581	0.423	3.239	4.398
	45	3.159	2.213	3.159	4.101	2.991	1.791	2.685	3.987	2.688	1.102	2.151	3.713
	30	4.101	3.159	2.213	3.159	3.987	2.685	1.791	2.991	3.712	2.120	1.102	2.688
	15	4.789	4.097	1.526	2.207	4.700	3.703	0.878	1.876	4.398	3.239	0.423	1.581
	0	5.041	4.750	1.271	1.971	4.901	4.540	0.678	1.084	4.539	4.115	0.700	0.276
45	-15	4.750	5.041	1.971	1.271	4.540	4.901	1.084	0.678	4.115	4.539	0.276	0.423
	-30	4.097	4.789	2.207	1.526	3.703	4.700	1.876	0.878	3.239	4.398	1.581	0.423
	-45	3.159	4.101	3.159	2.213	2.685	3.987	2.991	1.791	2.151	3.712	2.688	1.102
	-60	2.213	3.159	4.101	3.159	1.791	2.991	3.987	2.685	1.102	2.688	3.713	2.151
	-75	1.526	2.207	4.789	4.097	0.878	1.876	4.700	3.703	0.423	1.581	4.398	3.239
	60	2.114	1.279	2.591	3.426	1.946	0.781	2.155	3.320	1.706	0.404	1.681	2.983
	45	2.836	1.871	1.871	2.836	2.721	1.378	1.378	2.721	2.443	0.962	0.962	2.443
	30	3.426	2.591	1.279	2.114	3.320	2.155	0.781	1.946	2.983	1.681	0.404	1.706
	15	3.730	3.048	0.975	1.957	3.577	2.904	0.595	1.197	3.175	2.423	0.212	0.969
	0	3.848	3.848	1.040	1.040	3.425	3.425	0.677	0.677	2.971	2.971	0.417	0.417
	-15	3.548	3.730	1.957	0.975	2.904	3.577	1.197	0.595	2.423	3.175	0.969	0.212
	-30	2.591	3.426	2.114	1.279	2.155	3.320	1.946	0.781	1.681	2.983	1.706	0.404
	-45	1.871	2.836	2.836	1.871	1.378	2.721	2.721	1.378	0.962	2.443	2.443	0.962
	-60	1.279	2.114	3.426	2.591	0.781	1.946	3.320	2.155	0.404	1.706	2.983	1.681
	30	2.088	1.151	1.669	2.580	1.958	0.735	1.244	2.466	1.669	0.391	0.825	2.128
	15	2.580	1.669	1.151	2.088	2.466	1.244	0.735	1.958	2.128	0.825	0.391	1.669
	0	2.804	2.238	0.868	1.495	2.743	1.868	0.459	1.324	2.330	1.376	0.190	1.144
	-15	2.351	2.707	0.778	1.008	2.713	2.389	0.488	0.816	2.316	1.969	0.274	0.684
	-30	1.707	2.351	1.008	0.778	1.324	2.389	0.816	0.488	1.095	2.244	0.684	0.274
	-45	1.238	2.804	1.495	0.868	0.735	2.743	1.324	0.459	1.376	2.330	1.144	0.190
	-60	1.151	2.580	2.580	1.669	0.735	1.958	2.466	1.244	0.391	1.669	2.128	0.825
	45	2.000	1.080	1.080	2.000	1.886	0.708	0.708	1.886	1.560	0.377	0.377	1.560
	30	2.328	1.532	0.751	1.548	2.160	1.242	0.432	1.490	1.756	0.730	0.182	1.208
	15	2.442	1.983	0.437	1.096	2.205	1.617	0.387	0.975	1.761	1.148	0.199	0.781
	0	2.317	2.317	0.761	0.761	2.006	2.006	0.586	0.586	1.518	1.518	0.421	0.421
	-15	1.983	2.442	1.096	0.437	1.617	2.205	0.975	0.387	1.148	1.761	0.781	0.199
	-30	1.532	2.328	1.548	0.751	1.242	2.160	1.490	0.432	0.730	1.756	1.208	0.182
	-45	1.080	2.000	2.000	1.080	0.708	1.886	1.886	0.708	0.377	1.560	1.560	0.377
	75	1.998	1.033	0.719	1.574	1.728	0.687	0.420	1.463	1.367	0.361	0.181	1.167
	60	2.052	1.483	0.595	1.184	1.819	1.056	0.332	1.094	1.371	0.649	0.156	0.878
	45	2.012	1.788	0.396	0.806	1.710	1.490	0.441	0.720	1.133	0.969	0.296	0.590
	30	1.788	2.012	0.806	0.396	1.490	1.710	0.720	0.441	0.969	1.133	0.590	0.296
	-15	1.483	2.052	1.184	0.595	1.056	1.819	1.094	0.332	0.649	1.371	0.878	0.156
	-30	1.033	1.998	1.574	0.719	0.687	1.728	1.463	0.420	0.361	1.367	1.167	0.181
	30	1.738	0.997	0.511	1.250	1.512	0.666	0.303	1.149	1.091	0.343	0.133	0.881
	15	1.779	1.331	0.490	0.918	1.446	0.978	0.349	0.837	1.010	0.578	0.214	0.645
	0	1.610	1.610	0.639	0.639	1.271	1.271	0.544	0.544	0.823	0.823	0.402	0.402
	-15	1.331	1.779	0.918	0.490	0.978	1.446	0.837	0.349	0.578	1.010	0.645	0.214
	-30	0.997	1.738	1.250	0.511	0.666	1.512	1.149	0.303	0.343	1.091	0.881	0.133
	30	1.343	0.884	0.405	1.003	1.268	0.643	0.290	0.908	0.832	0.305	0.161	0.670
	15	1.458	1.246	0.509	0.781	1.131	0.904	0.423	0.648	0.815	0.515	0.293	0.478
	0	1.366	1.366	0.721	0.509	0.904	1.131	0.648	0.423	0.515	0.815	0.478	0.293
	-15	1.246	1.458	1.003	0.405	0.643	1.268	0.908	0.290	0.305	0.832	0.670	0.161
	15	1.319	0.931	0.498	0.808	1.005	0.618	0.335	0.722	0.599	0.305	0.216	0.510
	0	1.169	1.169	0.574	0.574	0.833	0.833	0.506	0.506	0.459	0.459	0.356	0.356
	-15	0.931	1.319	0.808	0.498	0.618	1.005	0.722	0.335	0.305	0.599	0.510	0.216
	15	1.083	0.894	0.466	0.693	0.765	0.589	0.401	0.576	0.408	0.285	0.267	0.390
	0	0.894	1.083	0.693	0.466	0.589	0.765	0.576	0.401	0.285	0.408	0.390	0.267
	0	0.894	0.894	0.595	0.595	0.558	0.558	0.460	0.460	0.265	0.265	0.298	0.298

Table 1. Stiffnesses for Circular Segments

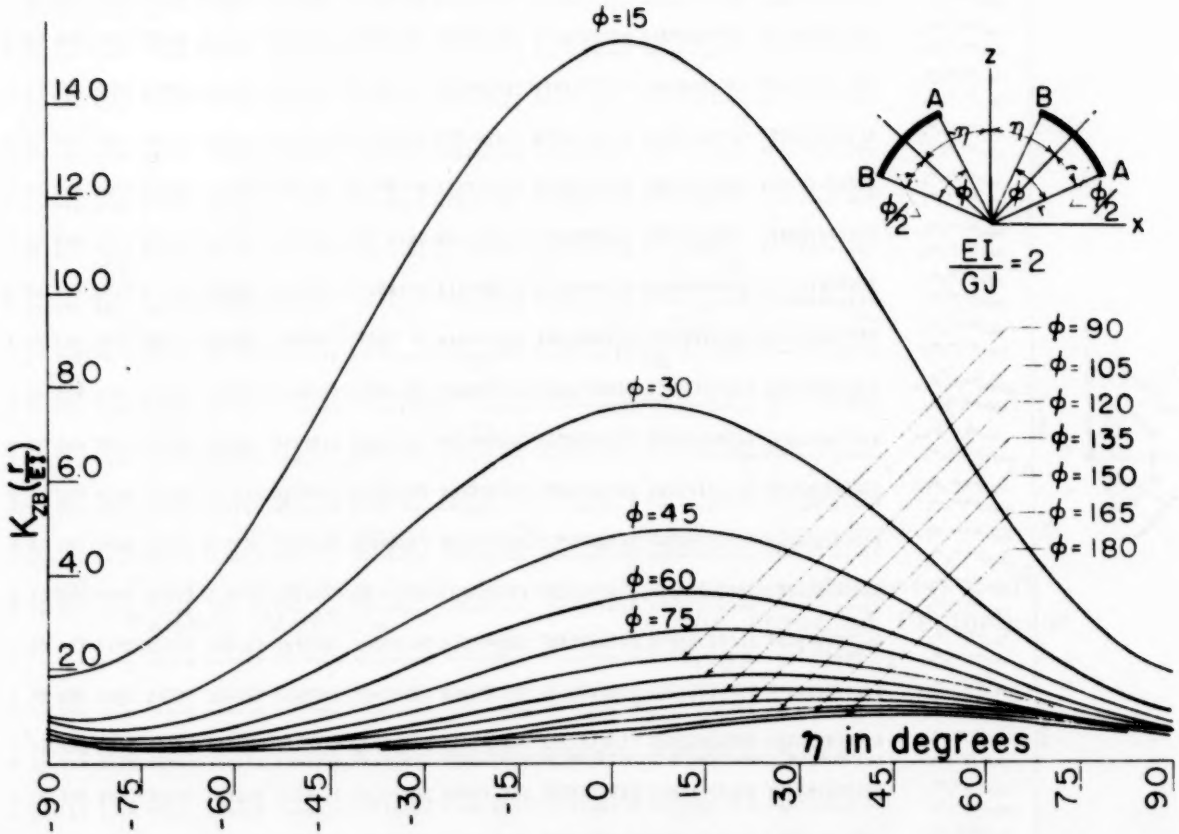


Fig. 3. Values of Stiffness for Circular Segments

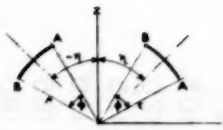
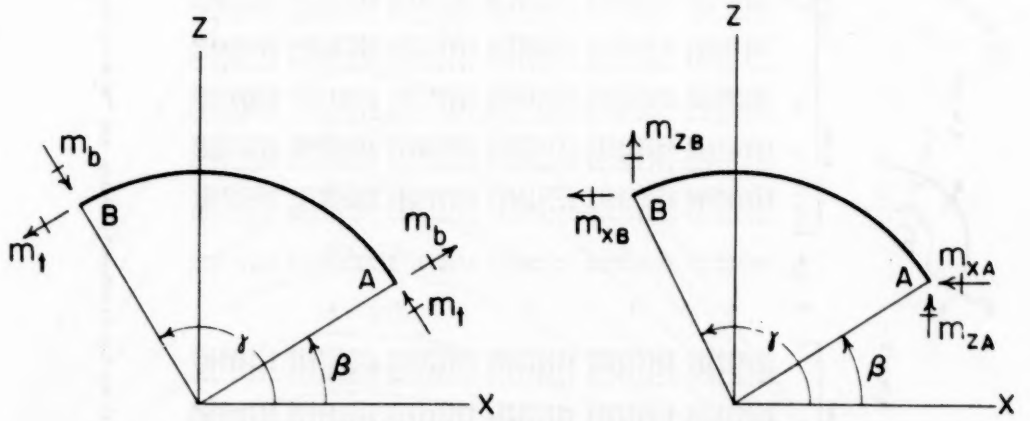


Table 2. Carry-over Factors (in terms of stiffnesses) for Circular Segments



(a) Positive Bending and Twisting Moments.

(b) Positive Moments about the Assumed Rectangular Axes x and z.

Fig. 4. Positive Moments at Ends of a Circular Segment

ϕ	t/r	m_A(1/kr)			m_B(1/kr)		
		α1/α2 = 1	2	6	1	2	6
	0.1	0.02123	0.02125	0.02128	0.00000	0.00000	0.00001
	0.2	0.03368	0.03378	0.03373	0.00000	0.00001	0.00002
	0.3	0.03760	0.03780	0.03777	0.00001	0.00001	0.00003
	0.4	0.03782	0.03786	0.03789	0.00001	0.00002	0.00004
15	0.5	0.03696	0.03691	0.03708	0.00001	0.00002	0.00004
	0.6	0.03567	0.03511	0.03551	0.00001	0.00002	0.00004
	0.7	0.01857	0.01660	0.01670	0.00001	0.00001	0.00001
	0.8	0.00836	0.00832	0.00845	0.00000	0.00000	0.00001
	0.9	0.00837	0.00836	0.00837	0.00000	0.00000	0.00000
	0.1	0.04068	0.04071	0.04094	0.00004	0.00005	0.00009
	0.2	0.06783	0.06788	0.06851	0.00009	0.00013	0.00020
	0.3	0.07798	0.07824	0.07932	0.00015	0.00020	0.00047
	0.4	0.07859	0.07880	0.07980	0.00019	0.00031	0.00059
30	0.5	0.06605	0.06689	0.06810	0.00019	0.00028	0.00061
	0.6	0.05115	0.05144	0.05236	0.00017	0.00029	0.00053
	0.7	0.01757	0.01375	0.01438	0.00012	0.00018	0.00039
	0.8	0.01704	0.01713	0.01743	0.00007	0.00010	0.00022
	0.9	0.00873	0.00878	0.00887	0.00001	0.00003	0.00006
	0.1	0.06438	0.06450	0.06509	0.00015	0.00021	0.00042
	0.2	0.10857	0.10916	0.10915	0.00045	0.00068	0.00127
	0.3	0.11880	0.11956	0.12061	0.00068	0.00106	0.00218
	0.4	0.11689	0.11791	0.12138	0.00093	0.00133	0.00263
45	0.5	0.10188	0.10298	0.10620	0.00097	0.00139	0.00278
	0.6	0.07838	0.07861	0.08157	0.00086	0.00129	0.00238
	0.7	0.05139	0.05058	0.05138	0.00063	0.00089	0.00173
	0.8	0.02805	0.02756	0.02686	0.00035	0.00049	0.00095
	0.9	0.00730	0.00716	0.00749	0.00010	0.00015	0.00028
	0.1	0.08884	0.08887	0.08881	0.00043	0.00063	0.00115
	0.2	0.13878	0.13997	0.14174	0.00138	0.00198	0.00330
	0.3	0.14430	0.14517	0.17256	0.00230	0.00319	0.00678
	0.4	0.13984	0.14158	0.14783	0.00250	0.00401	0.00717
60	0.5	0.11980	0.12119	0.14671	0.00202	0.00317	0.00736
	0.6	0.10719	0.10871	0.11278	0.00266	0.00367	0.00662
	0.7	0.07930	0.07923	0.08066	0.00195	0.00267	0.00460
	0.8	0.03758	0.03797	0.03870	0.00107	0.00147	0.00250
	0.9	0.00998	0.01002	0.01010	0.00032	0.00044	0.00074
	0.1	0.10809	0.10980	0.11188	0.00108	0.00149	0.00241
	0.2	0.17980	0.17884	0.18116	0.00331	0.00444	0.00710
	0.3	0.20689	0.20933	0.21767	0.00751	0.00736	0.01206
	0.4	0.20478	0.20812	0.21710	0.00699	0.00983	0.01499
75	0.5	0.17914	0.18071	0.18962	0.00789	0.00958	0.01524
	0.6	0.13501	0.14025	0.14491	0.00636	0.00840	0.01314
	0.7	0.09981	0.09168	0.09176	0.00443	0.00609	0.00938
	0.8	0.04566	0.04609	0.04639	0.00235	0.00313	0.00503
	0.9	0.01268	0.01275	0.01256	0.00076	0.00099	0.00146
	0.1	0.13230	0.13333	0.13614	0.00117	0.00158	0.00236
	0.2	0.21576	0.21860	0.22615	0.00668	0.00862	0.01310
	0.3	0.25380	0.25803	0.26869	0.01114	0.01430	0.02142
	0.4	0.25697	0.25795	0.26884	0.01403	0.01791	0.02617
90	0.5	0.22171	0.22569	0.23588	0.01468	0.01853	0.02677
	0.6	0.17079	0.17345	0.17783	0.01283	0.01619	0.02290
	0.7	0.11166	0.11285	0.11538	0.00938	0.01160	0.01612
	0.8	0.05611	0.05643	0.05550	0.00510	0.00629	0.00851
	0.9	0.01550	0.01547	0.01474	0.00151	0.00186	0.00240

ϕ	t/r	m_A(1/kr)			m_B(1/kr)		
		α1/α2 = 1	2	6	1	2	6
	0.1	0.15610	0.15743	0.16091	0.00189	0.00235	0.00329
	0.2	0.25671	0.25995	0.26949	0.01200	0.01411	0.02038
	0.3	0.30178	0.30905	0.32147	0.02004	0.02468	0.03409
	0.4	0.30461	0.30907	0.32099	0.02525	0.03084	0.04176
105	0.5	0.26671	0.27101	0.27887	0.02625	0.03184	0.04210
	0.6	0.20195	0.20787	0.21845	0.02274	0.02771	0.03645
	0.7	0.11375	0.11934	0.13111	0.01686	0.01985	0.02678
	0.8	0.04606	0.04807	0.04936	0.00907	0.01143	0.01288
	0.9	0.01838	0.01806	0.01856	0.00267	0.00311	0.00359
	0.1	0.18064	0.18085	0.18609	0.00640	0.00767	0.01028
	0.2	0.29988	0.30183	0.31193	0.01977	0.02354	0.03096
	0.3	0.35416	0.36203	0.37561	0.03309	0.03908	0.05032
	0.4	0.35716	0.36695	0.37976	0.04166	0.04898	0.06136
120	0.5	0.31365	0.32764	0.32411	0.04325	0.05017	0.06138
	0.6	0.24095	0.24249	0.24248	0.03783	0.04341	0.05147
	0.7	0.15600	0.15589	0.15139	0.02724	0.03090	0.03730
	0.8	0.07747	0.07841	0.07118	0.01474	0.01650	0.01802
	0.9	0.02101	0.02100	0.01775	0.00430	0.00470	0.00490
	0.1	0.20509	0.20711	0.21163	0.00981	0.01135	0.01419
	0.2	0.34313	0.34795	0.35951	0.03044	0.03490	0.04334
	0.3	0.41026	0.41848	0.43076	0.05103	0.05795	0.07029
	0.4	0.41292	0.41771	0.42917	0.06426	0.07324	0.08931
135	0.5	0.36168	0.36675	0.36920	0.06660	0.07602	0.09278
	0.6	0.27672	0.27684	0.27759	0.05805	0.06569	0.07938
	0.7	0.17829	0.17936	0.16887	0.04159	0.04698	0.04786
	0.8	0.08758	0.08930	0.07846	0.02274	0.02576	0.02385
	0.9	0.02345	0.02338	0.01895	0.00664	0.00673	0.00632
	0.1	0.23050	0.23251	0.23742	0.01426	0.01595	0.01930
	0.2	0.38798	0.39299	0.40512	0.04440	0.04917	0.05816
	0.3	0.46255	0.47188	0.48659	0.07455	0.08166	0.09421
	0.4	0.46804	0.47303	0.48581	0.09389	0.10360	0.11788
150	0.5	0.40969	0.41346	0.41380	0.09713	0.10373	0.11248
	0.6	0.31176	0.31000	0.30954	0.08433	0.08871	0.09245
	0.7	0.19914	0.19906	0.18980	0.06005	0.06210	0.06166
	0.8	0.09864	0.09837	0.08868	0.03198	0.03291	0.03089
	0.9	0.02567	0.02588	0.01953	0.00914	0.00904	0.00781
	0.1	0.25995	0.26216	0.26337	0.01908	0.02153	0.02502
	0.2	0.43383	0.43867	0.45144	0.06194	0.06651	0.07554
	0.3	0.52114	0.53185	0.54876	0.10419	0.11052	0.12797
	0.4	0.52395	0.52801	0.54897	0.13119	0.13730	0.14781
165	0.5	0.45611	0.45686	0.45796	0.13540	0.13958	0.14469
	0.6	0.34466	0.34112	0.34108	0.11702	0.11857	0.11539
	0.7	0.21775	0.21198	0.19779	0.08295	0.08219	0.07747
	0.8	0.10607	0.09906	0.08681	0.04365	0.04233	0.03708
	0.9	0.02688	0.02679	0.01976	0.01232	0.01161	0.00932
	0.1	0.26130	0.26395	0.26942	0.02023	0.02309	0.02857
	0.2	0.47850	0.48634	0.49758	0.08323	0.08706	0.09559
	0.3	0.57607	0.58314	0.59990	0.14005	0.14476	0.15479
	0.4	0.57710	0.58178	0.59822	0.17652	0.17950	0.18615
180	0.5	0.50000	0.50000	0.50000	0.18169	0.18169	0.18167
	0.6	0.37196	0.36987	0.35883	0.15621	0.15323	0.14659
	0.7	0.23295	0.22587	0.21011	0.10980	0.10509	0.09508
	0.8	0.10924	0.10324	0.08995	0.05716	0.05333	0.04481
	0.9	0.02750	0.02506	0.01959	0.01598	0.01433	0.01085

Table 3. Influence Values for Fixed-end Bending and Twisting Moments

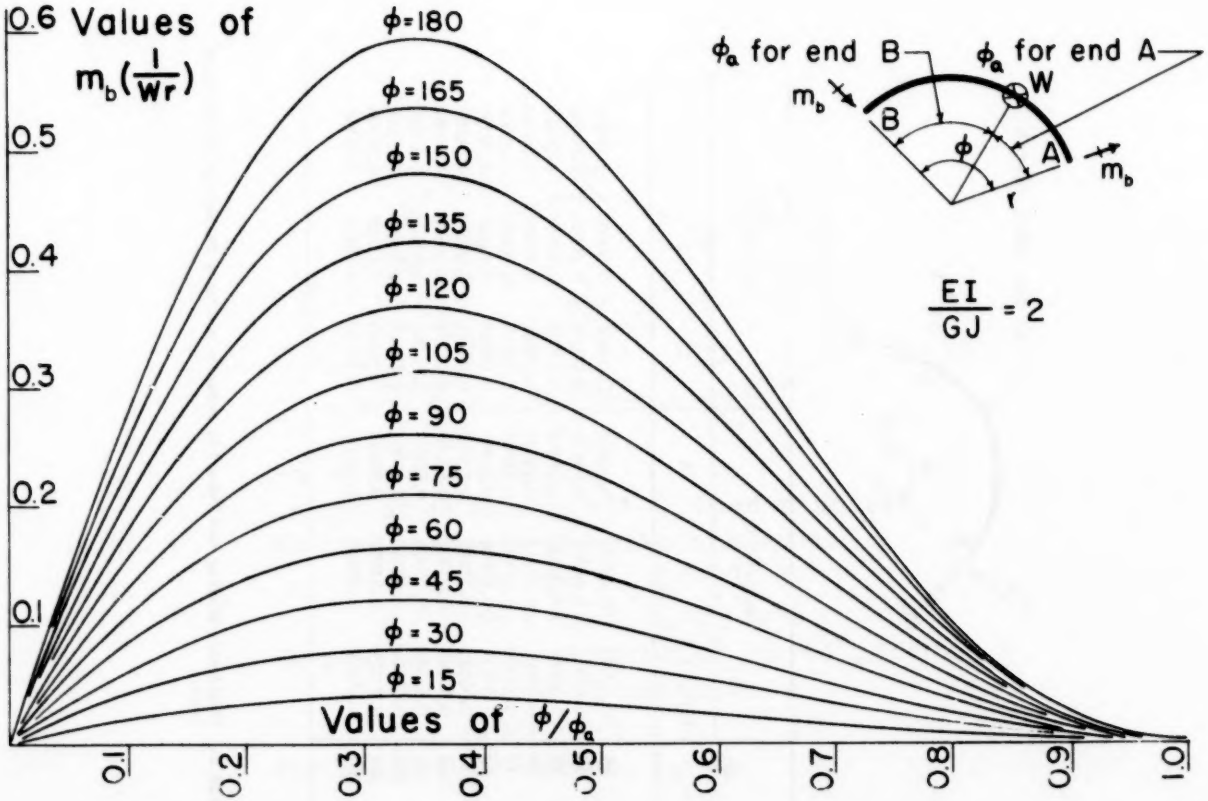
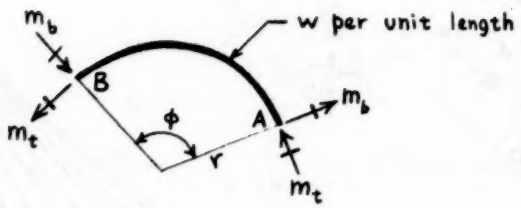
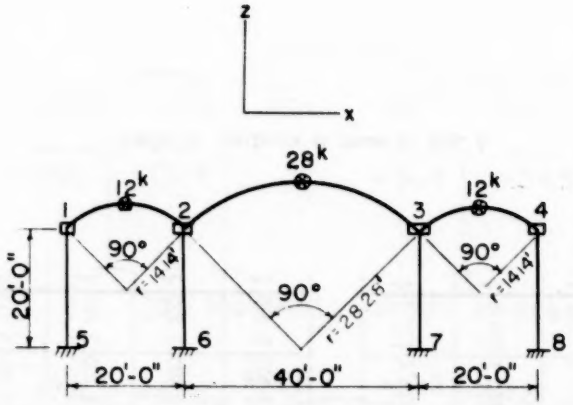


Fig. 5. Influence Lines for Fixed-end Bending Moments



ϕ	$m_b(1/wr^2)$			$m_t(1/wr^2)$		
	EI/GJ = 1	2	6	1	2	6
15	0.00573	0.00574	0.00576	0.00000	0.00000	0.00000
30	0.02315	0.02361	0.02521	0.00005	0.00012	0.00052
45	0.05292	0.05334	0.05469	0.00041	0.00058	0.00114
60	0.09601	0.09712	0.10022	0.00169	0.00233	0.00412
75	0.15358	0.15572	0.16088	0.00502	0.00666	0.01060
90	0.22676	0.23004	0.23790	0.01216	0.01543	0.02230
105	0.31643	0.32059	0.32822	0.02545	0.03087	0.04082
120	0.42301	0.42742	0.43458	0.04782	0.05546	0.06782
135	0.54624	0.55004	0.55557	0.08261	0.09180	0.10516
150	0.68502	0.68745	0.69057	0.13346	0.14253	0.15454
165	0.83729	0.83862	0.83913	0.20398	0.21028	0.21798
180	1.00000	1.00000	1.00000	0.29756	0.29756	0.29756

Table 4. Fixed-end Bending and Twisting Moments due to a Uniformly Distributed Load.



For all members, $EI/GJ = 1$
Length of all columns = 14'-0", fixed at bases
Relative moments of inertia of columns:-

cols. 1 and 4 : $I_x = 1$; $I_z = 3$
cols. 2 and 3 : $I_x = 2$; $I_z = 6$

Relative moments of inertia of curved girders:-

girders 1-2 and 3-4 : 4
girder 2-3 : 20

For all straight beams, $I_x = 3$

Fig. 6. Continuous Frame with Girders of Circular Arc Plan

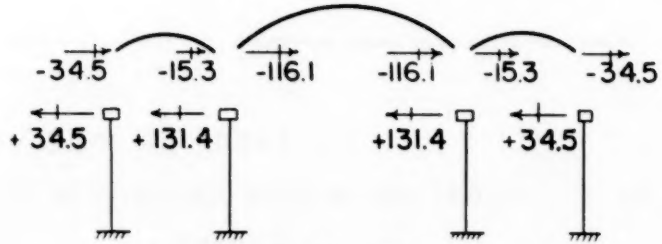
X - AXIS

Member	Joint 1	Joint 2		Joint 3		Joint 4
	1-2	2-1	2-3	3-2	3-4	4-3
Dist. Factors	0.195	0.122	0.280	0.280	0.122	0.195
C. O. Factors	$(C_{mm})_{2-2}$= -1.000 $(C_{ss})_{2-1}$= +0.601 $(C_{ss})_{1-1}$= +0.601	$(C_{mm})_{2-1}$= -1.000 $(C_{ss})_{2-3}$= +0.601 $(C_{ss})_{2-2}$= +0.601	$(C_{mm})_{2-3}$= -1.000 $(C_{ss})_{2-2}$= +0.601 $(C_{ss})_{3-3}$= +0.601	$(C_{mm})_{3-2}$= 1.000 $(C_{ss})_{3-3}$= -0.601 $(C_{ss})_{2-3}$= +0.601	$(C_{ss})_{3-4}$= -1.000 $(C_{ss})_{3-3}$= +0.601 $(C_{ss})_{4-4}$= +0.601	$(C_{mm})_{4-3}$= -1.000 $(C_{ss})_{4-4}$= +0.601 $(C_{ss})_{3-4}$= +0.601
F. E. M. Dist.	-38.9 + 4.9	-38.9 +15.8	-116.1 + 39.5	-116.1 + 39.5	-38.9 +15.8	-38.9 + 4.9
C. O. M.(s)	-15.8	- 4.8	- 39.5	- 39.5	- 4.8	-15.8
C. O. M.(s) C. O. M.(s) Dist.	+ 3.2 + 2.9 + 2.0	- 2.2 - 1.2 - 5.6	+ 8.1 + 8.1 + 11.9	+ 8.1 + 8.1 + 11.9	- 2.2 - 1.2 - 5.6	+ 3.2 + 2.9 + 2.0
C. O. M.(s)	- 5.6	+ 2.0	- 11.9	- 11.9	+ 2.0	- 5.6
C. O. M.(s) C. O. M.(s) Dist.	+ 0.6 - 0.1 + 1.0	+ 0.1 - 0.6 + 1.8	- 1.6 + 1.6 + 4.6	- 1.6 + 1.6 + 4.6	+ 0.1 - 0.6 + 1.8	+ 0.6 - 0.1 + 1.0
C. O. M.(s)	- 1.8	- 1.0	- 4.6	- 4.6	- 1.0	- 1.9
C. O. M.(s) C. O. M.(s) Dist.	+ 0.2 + 0.1 + 0.7	+ 0.1 - 0.2 + 0.7	- 0.5 + 0.5 + 1.7	- 0.5 + 0.5 + 1.7	+ 0.1 - 0.2 + 0.7	+ 0.2 + 0.1 + 0.2
C. O. M.(s)	+ 0.7	+ 0.2	- 1.7	- 1.7	+ 0.2	- 0.7
C. O. M.(s) C. O. M.(s) Dist.	0.0 0.0 + 0.1	0.0 0.0 + 0.1	- 0.1 + 0.1 + 0.6	- 0.1 + 0.1 + 0.6	0.0 0.0 + 0.1	0.0 0.0 + 0.1
Totals	-38.4	-15.2	-116.1	-116.1	-15.2	-38.4

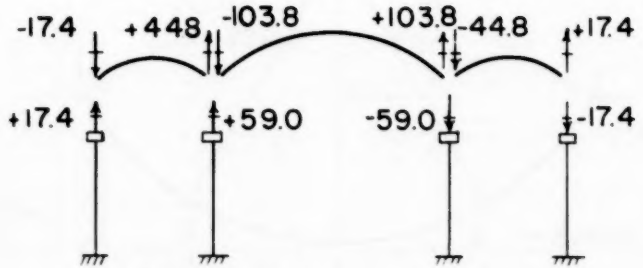
Y - AXIS

Joint 1	Joint 2		Joint 3		Joint 4
1-2	2-1	2-3	3-2	3-4	4-3
0.395	0.158	0.394	0.394	0.158	0.395
$(C_{ss})_{1-2}$= +0.652 $(C_{mm})_{1-1}$= -0.198 $(C_{mm})_{1-1}$= -0.198	$(C_{ss})_{2-1}$= +0.652 $(C_{mm})_{2-2}$= -0.198 $(C_{mm})_{2-2}$= -0.198	$(C_{ss})_{2-3}$= +0.652 $(C_{mm})_{2-3}$= -0.198 $(C_{mm})_{2-2}$= -0.198	$(C_{ss})_{3-2}$= +0.652 $(C_{mm})_{3-3}$= -0.198 $(C_{mm})_{3-3}$= -0.198	$(C_{ss})_{3-4}$= +0.652 $(C_{mm})_{3-4}$= -0.198 $(C_{mm})_{3-3}$= -0.198	$(C_{ss})_{4-3}$= +0.652 $(C_{mm})_{4-3}$= -0.198 $(C_{mm})_{3-4}$= -0.198
-38.4 +11.7	-38.4 +10.4	-132.3 + 46.9	+132.3 - 46.9	-38.4 +26.4	-38.4 +11.8
- 9.5 + 2.9 + 7.4	+ 2.9 - 9.5 + 7.1	+ 73.8 + 73.8 - 18.5	- 73.8 - 23.8 + 18.5	+ 2.9 - 9.5 + 7.1	- 9.5 + 2.9 - 7.4
- 8.3	- 1.7	- 7.8	- 7.8	- 1.7	+ 8.3
- 3.5 + 1.2 + 1.8	+ 1.2 - 3.5 - 0.1	- 8.1 + 8.1 - 3.5	+ 8.1 - 8.1 + 3.5	- 1.2 + 3.5 + 0.1	+ 3.5 - 1.2 - 1.8
+ 0.4	- 0.9	+ 2.1	- 2.1	+ 0.9	- 0.4
- 1.1 + 0.6 + 0.4	+ 0.6 - 1.1 + 0.2	- 2.8 + 2.8 - 1.0	+ 2.8 - 2.8 + 1.0	- 0.6 + 1.1 - 0.2	+ 1.1 - 0.6 - 0.4
0.0	+ 0.2	+ 0.5	- 0.5	+ 0.2	0.0
- 0.4 + 0.2 + 0.1	+ 0.2 - 0.4 0.0	- 1.0 + 1.0 - 0.2	+ 1.0 - 1.0 + 0.2	- 0.2 + 0.4 0.0	+ 0.4 - 0.2 - 0.1
0.0	+ 0.1	+ 0.2	- 0.2	+ 0.1	0.0
-17.4	+46.8	+103.8	-103.8	-46.8	+17.4

Table 5. Analysis of Frame of Fig. 6

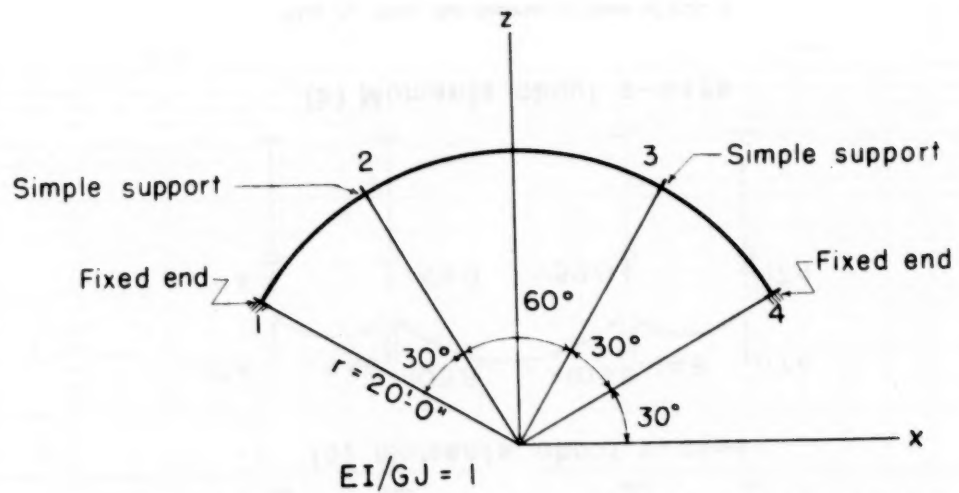


(a) Moments about x-axes



(b) Moments about z-axes

Fig. 7. Final End Moments in Frame of Fig. 6



EI/GJ = 1

Uniformly distributed load on all spans = 1 kip per ft.

Relative I for all spans = 1

Fig. 8. Continuous Girder of Circular Arc Plan

z - Axis

Member	Joint 1	Joint 2		Joint 3		Joint 4
	1-2	2-1	2-3	3-2	3-4	4-3
Dist. Factors	—	0.839	0.161	0.161	0.839	—
	—	$(C_{mz})_{2-1}$ = +0.375	$(C_{zz})_{2-3}$ = +1.000	$(C_{zz})_{3-2}$ = +1.000	$(C_{zz})_{3-4}$ = +0.375	—
C. O. Factors	—	$(C_{zz})_{2-1}$ = +0.669	$(C_{zz})_{2-3}$ = +0.995	$(C_{zz})_{3-2}$ = +0.995	$(C_{zz})_{3-4}$ = +0.669	—
	—	$(C_{zz})_{2-2}$ = +0.906	$(C_{zz})_{2-3}$ = +0.995	$(C_{zz})_{3-3}$ = +0.995	$(C_{zz})_{3-3}$ = +0.906	—
F. E. M.	- 8.16	+ 4.76	-37.24	-37.24	+ 4.76	- 8.16
Dist.		+27.36	+ 5.22	+ 5.22	+27.36	
C. O. M.(z)	+10.71		- 5.22	- 5.22		+10.71
C. O. M.(z)	+22.15	+29.65	- 4.82	- 4.82	+29.65	+22.15
			+ 4.82	+ 4.82		
Dist.		-11.62	- 2.81	- 2.81	-11.62	
C. O. M.(z)	- 5.50		+ 2.81	+ 2.81		- 5.50
C. O. M.(z)	- 4.75	- 4.86	+ 1.03	+ 1.03	- 4.86	- 4.75
			- 1.03	- 1.03		
Dist.		+ 1.72	+ 0.33	+ 0.33	+ 1.72	
C. O. M.(z)	+ 0.65		- 0.33	- 0.33		+ 0.65
C. O. M.(z)	+ 3.96	+ 4.06	- 0.86	- 0.86	+ 4.06	+ 3.96
			+ 0.86	+ 0.86		
Dist.		- 3.13	- 0.60	- 0.60	- 3.13	
C. O. M.(z)	- 1.18		+ 0.60	+ 0.60		- 1.18
C. O. M.(z)	+ 0.21	+ 0.22	+ 0.05	+ 0.05	+ 0.22	+ 0.21
			+ 0.05	+ 0.05		
Dist.		- 0.69	- 0.13	- 0.13	- 0.69	
C. O. M.(z)	- 0.26		+ 0.13	+ 0.13		- 0.26
C. O. M.(z)	+ 1.09	+ 1.12	- 0.24	- 0.24	+ 1.12	+ 1.09
			+ 0.24	+ 0.24		
Dist.		- 1.05	- 0.20	- 0.20	- 1.05	
C. O. M.(z)	- 0.39		+ 0.20	+ 0.20		- 0.39
C. O. M.(z)	+ 0.65	+ 0.66	- 0.10	- 0.10	+ 0.66	+ 0.65
			+ 0.10	+ 0.10		
Dist.		- 0.55	- 0.11	- 0.11	- 0.55	
Totals	+18.68	+37.74	-37.24	-37.24	+37.24	+18.68

x - Axis

Joint 1	Joint 2		Joint 3		Joint 4
1-2	2-1	2-3	3-2	3-4	4-3
—	0.591	0.409	0.409	0.591	—
—	$(C_{zz})_{2-1}$ = +0.265	$(C_{zz})_{2-3}$ = +0.687	$(C_{zz})_{3-2}$ = 0.687	$(C_{zz})_{3-4}$ = +0.265	—
—	$(C_{mz})_{2-1}$ = +0.625	$(C_{mz})_{2-3}$ = +0.197	$(C_{mz})_{3-2}$ = +0.197	$(C_{mz})_{3-4}$ = +0.625	—
—	$(C_{zz})_{2-2}$ = +0.660	$(C_{zz})_{2-2}$ = +0.197	$(C_{zz})_{3-3}$ = +0.197	$(C_{zz})_{3-3}$ = +0.660	—
- 4.76	+ 8.16	-68.10	-68.10	- 8.16	+ 4.76
	+75.46	+28.50	+28.50	-75.46	
+12.78	+28.73	- 5.20	+ 5.20	-28.73	-12.78
		+ 5.20	- 5.20		
+ 9.38		-11.90	+11.90		- 9.38
	- 7.60	- 5.23	+ 5.23	+ 7.60	
- 6.86	-13.25	+ 2.80	- 2.80	+13.25	+ 6.86
		+ 2.80	+ 2.80		
- 2.01		+ 2.55	- 2.55		+ 2.01
	+ 6.33	+ 4.37	- 4.37	- 6.33	
+ 0.81	+ 1.56	+ 0.33	+ 0.33	- 1.56	- 0.81
		+ 0.33	+ 0.33		
+ 1.66		+ 2.13	+ 2.13		- 1.66
	+ 0.36	+ 0.23	- 0.23	- 0.36	
- 1.47	- 2.84	+ 0.60	- 0.60	+ 2.84	+ 1.47
		+ 0.60	+ 0.60		
+ 0.09		+ 0.11	+ 0.11		- 0.09
	+ 1.75	+ 1.20	- 1.20	- 1.75	
- 0.32	- 0.63	+ 0.13	- 0.13	+ 0.63	+ 0.32
		+ 0.13	+ 0.13		
+ 0.66		+ 0.58	+ 0.58		- 0.66
	+ 0.72	+ 0.49	- 0.49	- 0.72	
- 0.49	- 0.95	+ 0.20	+ 0.20	+ 0.95	+ 0.49
		+ 0.20	+ 0.20		
+ 0.19		+ 0.34	+ 0.34		- 0.19
	+ 0.70	+ 0.49	- 0.49	- 0.70	
+ 9.38	+82.46	-82.46	+82.46	-82.46	- 9.38

Table 6. Analysis of Continuous Curved Girder of Fig. 8

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